Distributivity and $(-1)x = -x$

Proposed by T. J. Kaczynski, Evergreen Park, Illinois

Let K be an algebraic system with two binary operations (one written additively, the other multiplicatively), satisfying:
(1) K is an abelian group under addition,
(2) $K - \{0\}$ is a group under multiplication, and
(3) $x(y + z) = xy + xz$ for all $x, y, z \in K$.
Suppose that for some n, $0 = 1 + 1 + \ldots + 1$ (n times). Prove that, for all $x \in K$, $(-1)x = -x$.

Solution by R.G. Bilyeu, North Texas State University. The last part of the hypothesis is unnecessary. If z denotes -1, then $z + z + zz = z(1 + 1 + z) = z$, so $z(?) = 1$. Now $z(x + zx) = zx + x = x + zx$, so either $x + zx = 0$ or $z = 1$. In either case $z(?) = -x$.

Also solved by Carol Avelsgaard, Richard Bourgin, Robert Bowen, Joel Brawley, Jr., F. P. Callahan, M. M. Chawla (India), R. A. Cunninghame-Green (England), M. J. DeLeon, M. Edelstein, N. J. Fine, Harvey Friedman, Anton Glaser, M. G. Greening (Australia), A. G. Heinicke, Sidney Heller, G. A. Heuer, Stephen Hoffman, K. G. Johnson, A. J. Karson, Max Klicker, Kwangil Koh, C. C. Lindner, C. R. MacCluer, H. F. Mattson, C. J. Maxson, R. V. Moddy, José Morgado (Brazil), W. L. Owen, Jr., P. R. Parthasarathy (India), Harsh Pittie, Kenneth Rogers, Tôru Saitô (Japan), Camilio Schmidt, Leonard Shapiro, Frank A. Smith, George Van Zwalenberg, W. C. Waterhouse, Kenneth Yanosko, and the proposer.